

The Impact of a Flat Plate on a Water Surface

By J. H. G. Verhagen¹

The phenomena occurring during the impact of a flat plate on a water surface are investigated both theoretically and experimentally. It is shown that, by taking the influence of the compressible air layer caught between the falling plate and the water surface properly into account, the experimental results can be fully explained. Numerical results showing the effects of falling mass, drop velocity, and plate dimensions on the maximum impact pressure are available.

Introduction

DURING the past decades, many theoretical and experimental studies have been made to clarify the basic properties of slamming and to correlate slamming pressures with the section form of a ship's hull.

In most theoretical treatments of the ship slamming problem the water is considered to be incompressible and nonviscous. In the mathematical idealization of the problem it is assumed, moreover, that the velocity of a specified part of the free water surface is increased instantaneously from zero to the downward speed of the falling body. The slamming pressures resulting from such analyses are in good agreement with experimental results as long as the deadrise angle of the ship's bottom is not too small (see e.g. Wagner [1]²). In the extreme case of a flat bottom, however, the theory predicts infinitely high pressures at the instant of impact. Therefore, some buffer device should be developed to explain the discrepancy between theory and experiment. For this purpose, the compressibility of the water can be taken into account (von Kármán [2], Ogilvie [3], Egorov [4], et al.). The initial pressure at the instant of impact resulting from such treatments appears to be independent of the plate dimensions and equal to the acoustic pressure $p = \rho_l c_l V$, where ρ_l is the density of water, c_l the velocity of sound propagation in water, and V the downward body speed at impact. The duration of the pressure peak is in the order of $t = l/c_l$, where l is the half width of the flat bottom.

Sharov [5] considered also the influence of the elasticity of the construction on slamming pressures. He concluded from his investigation that the initial pressure is equal to $\rho_l c_l V$, independent of the elastic properties of the construction.

The high pressures predicted by these theories could not be verified experimentally. Measurements on this subject show a much lower impact pressure acting, moreover, during a longer time. Fujita [6] and Tanaka [7] considered the influence of the air layer caught between the flat plate and the water surface on the slamming pressure. Tanaka assumed that the pressure of the viscous air layer between a circular plate and the water causes a depression in the water surface. The plate would touch the water annularly from its circumference to the center. He carried out his calculations in the same way as Wagner's, modified with some of his own assumptions.

Fujita believes that the impact of a flat-bottom body is due to the pressure increase caused by the rapid compression of the air layer between bottom and water surface. He found the results obtained from such an assumption in good qualitative agreement with the experiments.

In order to be able to develop a more realistic theoretical model, it was deemed necessary to have a clearer picture of the phenomena occurring at the moment of impact. Experiments with different impact velocities, therefore, were carried out on a rigid flat rectangular plate slamming on a smooth water surface. The phenomena were filmed with a high-speed camera (8000 frames/sec). The pressure was recorded as a function of time. The pictures show that the water elevation is at first noticeable at the plate edges just before the plate touches the water surface. Next, the air layer breaks up into bubbles beginning at the edges and extending to the center of the plate with a speed in the order of magnitude

¹ Netherlands Ship Model Basin, Wageningen, The Netherlands.

² Numbers in brackets designate References at end of paper.

Manuscript received at SNAME Headquarters, June 13, 1966.
Revised manuscript received April 24, 1967.

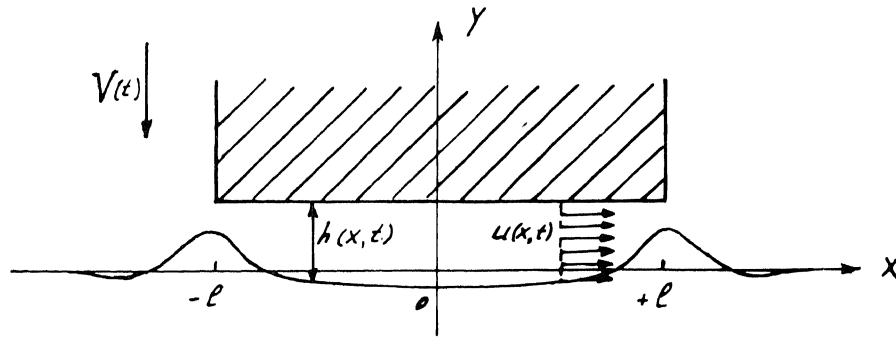


Fig. 1

equal to the velocity of sound in air. With increasing time the bubbles drift relatively slowly outwards.

A theory is presented to calculate the pressure as a function of time during impact.

Considering the flow in the air layer under the falling body to be a one-dimensional compressible flow of a perfect gas, the flow conditions are calculated by the method of characteristics. The disturbance of the water surface due to the imposed air pressure is solved with the aid of potential theory. Compressibility of the water is neglected, as the characteristic time interval for the impulsive pressure appears to be large compared to l/c_a . At the moment the edges of the plate touch the upheaved water surface an air cushion is trapped. This volume of air is calculated. Assuming that the escape of air after that moment can be neglected and that the downward velocity of the mean water surface under the plate is independent of the place coordinate, the adiabatic compression of the air layer is calculated.

Numerical results are compared with experimental results. During these investigations a DTMB report has been published by Sheng-Lun Chuang [8] describing similar experiments. His results and the data reported here show a general agreement.

Formulation of the Problem

A two-dimensional plate with width $2l$ is travelling downward at a velocity $V(t)$. The flat surface of the plate remains parallel to the undisturbed free water surface, Fig. 1. The free surface is taken initially as the x -axis and its shape is described by the equation

$$y - Y(x, t) = 0 \quad (1)$$

Basic Equations for the Air Layer

The problem is symmetrical about the y -axis.

The air between the plate and the water surface is driven away from the center to the edges of the plate. The pressure in the air layer increases with decreasing distance $h(x, t)$ between the plate and the water surface. As we are especially interested in that period of time where the pressure increases reasonably, i.e. for values

of $h/l \ll 1$, the flow in the air layer is considered as one-dimensional.

Neglecting viscosity effects, the equation of motion of the air is

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = - \frac{1}{\rho} \frac{\partial p}{\partial x} \quad (2)$$

The continuity equation becomes

$$\frac{\partial(\rho h)}{\partial t} + \frac{\partial(\rho u h)}{\partial x} = 0 \quad (3)$$

where

$u(x, t)$ = velocity of air in horizontal direction

$p(x, t)$ = pressure force

$\rho(x, t)$ = density of air

Assuming the compression of air to be isentropic

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0} \right)^\gamma \quad (4)$$

and using the equation

$$\frac{dp}{d\rho} = c_a^2 \quad (5)$$

where c_a is the velocity of sound in air, the equations of motion and continuity can be written as:

$$\left\{ \frac{\partial}{\partial t} + (u + c_a) \frac{\partial}{\partial x} \right\} \left(u + \frac{2c_a}{\gamma - 1} \right) = - \frac{c_a}{h} \left\{ \frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} \right\} \quad (6)$$

$$\left\{ \frac{\partial}{\partial t} + (u - c_a) \frac{\partial}{\partial x} \right\} \left(u - \frac{2c_a}{\gamma - 1} \right) = \frac{c_a}{h} \left\{ \frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} \right\}$$

Apart from this system of partial differential equations of hyperbolic type, a third relation is needed for the three unknown variables $u(x, t)$, $c_a(x, t)$ and $h(x, t)$.

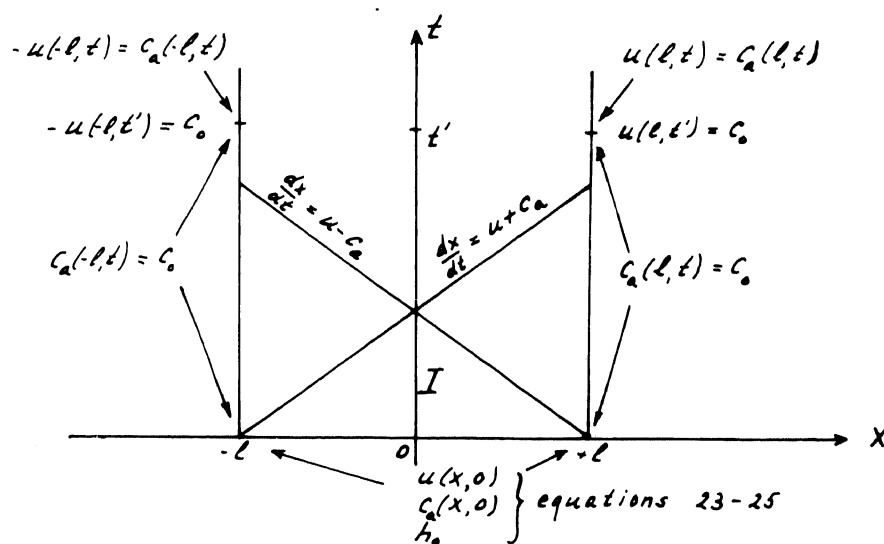


Fig. 2

The relation between the thickness of the air layer and the water elevation is given by

$$-\frac{\partial h}{\partial t} = V(t) + \frac{\partial Y(x, t)}{\partial t} \quad (7)$$

The downward velocity $V(t)$ can be a prescribed function of time, or in the case of a body free falling under gravity it can be derived from the momentum equation acting on the plate. With the initial condition $V = 0$ for $t = t_0$ the downward velocity is

$$V(t) = g(t - t_0) + \frac{1}{M} \int_{t_0}^t dt \int_{-l}^l (p - p_0) dx \quad (8)$$

where M is the mass of the flat-bottomed body per unit length and g is the acceleration of gravity.

Since the pressure is on account of (4) and (5) related to the sound velocity by

$$\frac{p}{p_0} = \left(\frac{c_a}{c_0}\right)^{\frac{2\gamma}{\gamma-1}} \quad (9)$$

a relation between the pressure distribution in the air layer and the water elevation $Y(x, t)$ suffices to determine the motion completely.

Wave Motions Resulting from a Pressure Distribution Suddenly Applied to the Water Surface

Measurements on the pressure-time history occurring during the impact of a flat-bottomed body on the water surface show that the maximum impact pressure is only a few kilograms per square centimeter up to impact velocities of several meters per second.

The duration of the pressure impulse is long compared with the time required by an acoustic disturbance in the water to travel a distance equal to the half width of the flat bottom. These observations suggest that it is hardly likely that the compressibility of the water is playing an

important role on slamming phenomena. Therefore, the assumption is made that the water can be treated as an incompressible fluid. Assuming further nonviscous and irrotational flow, a velocity potential can be introduced. This velocity potential has to satisfy the Laplace equation as well as boundary conditions on the free surface. For $t < 0$ the water is undisturbed, and starting with $t = 0$ the pressure over the free surface is a given function $p(x, t)$. Assuming the disturbances of the water surface to be small, the linearized free-surface conditions can be used

$$Y_t - \varphi_y = 0$$

on the plane $y = 0$

$$\varphi_t + gY + \frac{p(x, t)}{\rho_l} = 0 \quad (10)$$

where ρ_l is the mass density of the liquid

The solution of this problem is given by Wehausen and Laitone [9]. The complex velocity potential is

$$f(z, t) = -\frac{1}{\pi \rho_l} \int_{-\infty}^{\infty} d\xi \int_0^t p(\xi, \tau) d\tau \int_0^{\infty} \cos \{(gk)^{1/2}(\tau - t)\} e^{-ik(z - \xi)} dk \quad (11)$$

where $z = x + iy$ and $y < 0$

The velocity of the water surface follows from

$$\frac{\partial}{\partial t} Y(x, t) = \frac{\partial}{\partial y} \varphi(x, 0, t) = -\lim_{y \rightarrow 0} \operatorname{Im} \frac{df}{dz} \quad (12)$$

which can be integrated to yield $Y(x, t)$.

The water surface elevation has to be calculated from $t = 0$ till the time t_c , at which there is a first contact between some point of the falling body and the water surface.

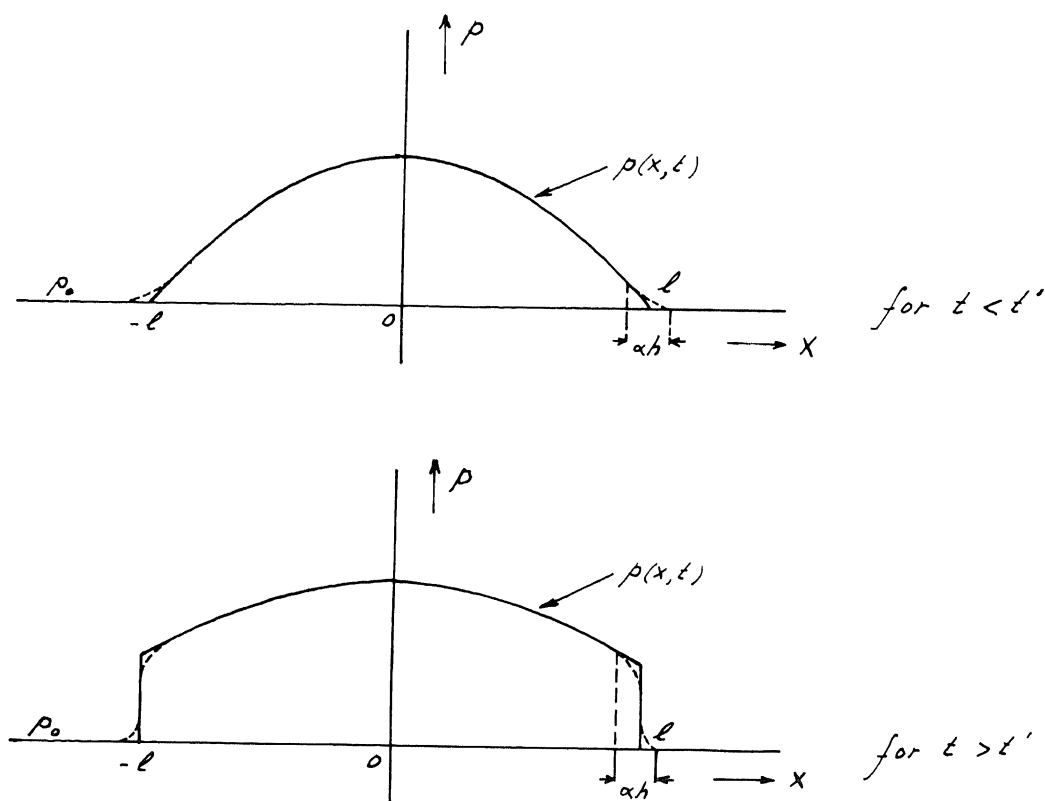


Fig. 3

From experimental observation it can be deduced that

$$\frac{\rho_l g Y(x, t)}{p(x, t)}$$

must be small compared to unity for the largest part of the region $|x| < l$ and for $0 < t < t_c$. On account of this observation and in order to simplify the calculations the gravity term is neglected. Equation (11) reduces then to

$$f(z, t) = -\frac{1}{\pi \rho_l} \int_0^t d\tau \int_{-\infty}^{\infty} \frac{i p(\xi, \tau)}{z - \xi} d\xi \quad \text{for } y < 0 \quad (13)$$

and equation (12) becomes

$$\frac{\partial}{\partial t} Y(x, t) = -\lim_{u \rightarrow 0} \frac{1}{\pi \rho_l} \operatorname{Im} \int_0^t d\tau \int_{-\infty}^{\infty} \frac{i p(\xi, \tau)}{(z - \xi)^2} d\xi \quad (14)$$

With the condition that $p(x, t)$ is zero at infinity, an integration by parts yields

$$\frac{\partial Y(x, t)}{\partial t} = -\frac{1}{\pi \rho_l} \int_0^t d\tau \int_{-\infty}^{\infty} \frac{\frac{\partial p}{\partial \xi}(\xi, \tau)}{x - \xi} d\xi \quad (15)$$

Thus, the elevation of the water surface becomes

$$Y(x, t) = -\frac{1}{\pi \rho_l} \int_0^t d\tau \int_0^{\tau} d\tau' \int_{-\infty}^{\infty} \frac{\frac{\partial p}{\partial \xi}(\xi, \tau')}{x - \xi} d\xi \quad (16)$$

Due to the condition $p(x, t) = 0$ for $|x| > l$ the integration over ξ from $-\infty$ to $+\infty$ can be reduced to the interval $-l \leq \xi \leq l$.

It may be observed from equation (16) that $Y(\pm l, t)$ will tend to infinite when $\partial p / \partial x$ does not equal zero at the points $x = \pm l$.

Solution of the Aerodynamic Part of the Problem

We have to solve the following system of partial differential equations for the unknown functions $u(x, t)$, $c_a(x, t)$, and $h(x, t)$.

$$\left\{ \frac{\partial}{\partial t} + (u + c_a) \frac{\partial}{\partial x} \right\} \left(u + \frac{2c_a}{\gamma - 1} \right) = -\frac{c_a}{h} \left\{ \frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} \right\} \quad (17)$$

$$\left\{ \frac{\partial}{\partial t} + (u - c_a) \frac{\partial}{\partial x} \right\} \left(u - \frac{2c_a}{\gamma - 1} \right) = \frac{c_a}{h} \left\{ \frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} \right\} \quad (18)$$

A third relation between $h(x, t)$ and $c_a(x, t)$ can be obtained by integrating relation (7). With the use of equations (16) and (9) the result is

$$\begin{aligned}
h(x,t) &= h_0 - Y(x,t) - \int_0^t V dt = \\
&= h_0 + \frac{p_0}{\pi \rho_0} \int_0^t d\tau \int_0^\tau d\tau' \int_{-\infty}^\infty \frac{\frac{d}{d\xi} \left(\frac{c_a}{c_0} \right)^{\frac{2\gamma}{\gamma-1}}}{x - \xi} d\xi - \int_0^t V dt
\end{aligned} \quad (19)$$

where h_0 is the position of the plate at time $t = 0$, while $t = 0$ is a rather arbitrary starting point of the calculation.

Initial and Boundary Conditions

To determine the solution of the foregoing hyperbolic differential equations, prescribed values of u , c_a , and h along a curve in the x - t plane are needed.

Due to the formulation of the problem it must be assumed that the validity of the derived equations is restricted to that region of the x - t plane for which $|x| \leq l$. The time interval is also restricted by the condition that h/l must be small compared to unity.

At first glance, it seems obvious to make the following simple choice of the initial conditions on the line segment $t = 0$, $|x| \leq l$:

$$h(x,0) = h_0, u(x,0) = 0, c_a(x,0) = c_0$$

where c_0 is the acoustic speed of sound in air. The last condition implies that the pressure under the plate is equal to the undisturbed pressure in the surrounding medium. However, there are some objections against these particular conditions. First, the question can be raised whether the final result will be affected by the arbitrary choice of the initial conditions. Secondly, these conditions lead to the appearance of a shock wave in the air layer, travelling from the edges of the plate to its center, with the acoustic speed c_0 . The pressure distribution on the water surface contains now a discontinuity due to the pressure jump across the shock wave, which results in an infinite water surface elevation according to equation (16).

To avoid mathematical difficulties and to get a more realistic picture of the physical phenomena, another choice of the initial conditions is made.

If the velocity of the air under the plate is small compared with the velocity of sound c_0 , it may be assumed that the air is incompressible and the water surface undisturbed. The air velocity, which follows from the continuity equation

$$\frac{dh}{dt} + h \frac{\partial u}{\partial x} = 0$$

with $u(0,t) = 0$, is

$$u(x,t) = -\frac{x}{h} \frac{dh}{dt} \quad (20)$$

and the pressure distribution, from

$$-\frac{1}{\rho_0} \frac{\partial p}{\partial x} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x}$$

with $p(\pm l, t) = p_0$, is

$$p(x,t) - p_0 = \rho_0 \frac{l^2 - x^2}{h^2} \left\{ \left(\frac{dh}{dt} \right)^2 - \frac{h}{2} \frac{d^2 h}{dt^2} \right\} \quad (21)$$

where

$$h(t) = h_0 - \int_0^t V(t) dt \quad (22)$$

With the downward velocity $V(t)$ of the plate given, the flow conditions can be calculated.

The choice of h_0 is restricted by the conditions that $h_0/l \ll 1$ and $u(l, t)/c_0 \ll 1$, so the initial conditions for $-l \leq x \leq +l$, $t = 0$ can be written as

$$\frac{V_0}{c_0} \ll \frac{h_0}{l} \ll 1; \quad Y(x,0) = 0 \quad (23)$$

$$\frac{u(x,0)}{c_0} = \frac{x V_0}{h_0 c_0} \quad (24)$$

$$\begin{aligned}
\frac{c_a(x,0)}{c_0} &= \left(\frac{p(x,0)}{p_0} \right)^{\frac{\gamma-1}{2\gamma}} \\
&= \left[1 + \gamma \frac{l^2 - x^2}{h_0^2} \right. \\
&\quad \times \left. \left\{ \frac{V_0^2}{c_0^2} + \frac{h_0}{2c_0^2} \left(\frac{dV}{dt} \right)_{t=0} \right\} \right]^{\frac{\gamma-1}{2\gamma}} \quad (25)
\end{aligned}$$

$$\begin{aligned}
&\approx 1 + \frac{\gamma-1}{2} \frac{l^2}{h_0^2} \frac{V_0^2}{c_0^2} \left(1 - \frac{x^2}{l^2} \right) \\
&\times \left\{ 1 + \frac{1}{2} \frac{h_0}{V_0^2} \left(\frac{dV}{dt} \right)_{t=0} \right\} + O \left(\frac{l V_0}{h_0 c_0} \right)^4
\end{aligned}$$

With these initial conditions no shock wave in the air layer will appear. The initial excess pressure under the plate is small enough to justify the assumption that the water surface is undisturbed. This implies that the effect of the choice of h_0 on the final result can be neglected as well. (N.B. h_0 is the position of the plate at the starting point of the numerical calculation; it has nothing to do with the total drop height of the plate.)

The solution of equations (17-19) can be determined now in the region I of the x - t plane bounded by two characteristics, defined by $dx/dt = u \pm c_a$ and the line segment $t = 0$, $-l \leq x \leq +l$ (see Fig. 2).

In order to extend the solution to larger values of time, boundary conditions for $x = \pm l$ are needed.

It seems obvious to assume that $p(\pm l, t) = p_0$ and hence $c(\pm l, t) = c_0$ as long as the escape velocity of the air at the edges of the plate does not exceed the acoustic sound velocity c_0 .

Let t' be the time at which the escape velocity of the air attains the value c_0 . For $t > t'$ the escape velocity at the plate edges will be equal to the local sound velocity $c_a(l, t)$. So the boundary conditions are:

for $0 \leq t \leq t'$

$$c_a(l, t) = c_a(-l, t) = c_0 \quad (26)$$

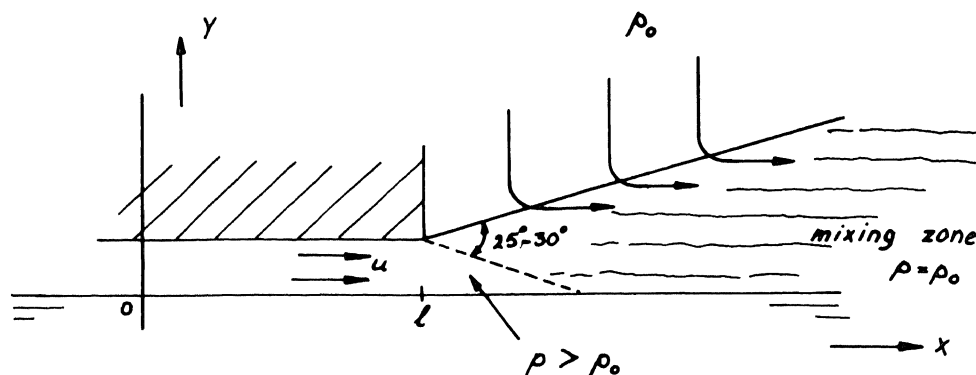


Fig. 4

for $t \geq t'$

$$\begin{aligned} u(l, t) &= c_a(l, t) \\ -u(-l, t) &= c_a(-l, t) \end{aligned} \quad (27)$$

where the value of t' is defined by

$$u(l, t') = c_0 \quad (28)$$

To avoid inconvenient singularities in the mathematical description of the problem, the conditions at the edges of the plate need a slight modification, as will be discussed in the next section.

Flow Condition at the Edges of the Plate

In the previous sections use was made of the statement that the pressure on the linearized free water surface immediately outside the plate edges is equal to the undisturbed pressure prevailing in the surrounding medium, so

$$p(x, y, t)|_{y=0} = p_0 \quad \text{for } |x| > l$$

When the undisturbed water surface is replaced by a rigid bottom and the pressure under the plate is calculated with the use of the boundary conditions (26–28), the result will be as given in Fig. 3. The first derivative of the pressure distribution in the x -direction will in the first case show a jump and in the second case a delta function at the edges of the plate. According to equation (16) the water elevation should have a logarithmic singularity for $t < t'$ and a $1/x$ singularity for $t > t'$. Actually the pressure distribution will be continuous at the edges of the plate. The used approximation of one-dimensional flow in the air layer is too rough at the end points. The only conclusion that can be drawn from the foregoing is that the acceleration of the water surface at the plate edges is much more important for $t > t'$ than for smaller values of time. Consider the air flow of the jet issuing from the slit between the edges of the plate and the water surface more closely. The Reynolds number uh/ν with which the plane jet enters into the surrounding medium is in practical cases sufficiently large to ensure the jet to be fully turbulent. Suppose the water surface remains undis-

turbed. The structure of the jet may be sketched as in Fig. 4. Near the slit, the jet boundary is diffused into a highly turbulent plane "mixing zone" with wedge-shaped profile. This gradually "eats" into the undisturbed central core of the jet. The angle of spread of the mixing zone ranges from 25–30 deg, and causes the nonturbulent cone of the jet to be eaten up after four or five times the slit opening [10]. At the boundaries of the jet the pressure equals p_0 . Inside the cone on the line $y = 0$ the pressure will gradually decrease in the x -direction to the value p_0 .

In view of this explanation it is assumed that the pressure curve as a function of x as shown in Fig. 3 must be smoothed at the plate edges over a region proportional to h , say αh where α is of the order of unity.

Owing to this revised pressure distribution, the singularities in the water elevation at the plate edges are transformed into finite peak values. From equation (16) together with the conditions as stated in the foregoing, the maximum surface elevation can be approximated by

$$Y(\pm l, t) = \frac{1}{\pi \rho l} \int_0^t d\tau \int_0^\tau \ln \left(\frac{\alpha h}{l} \right) \frac{\partial p(l, 0, \tau')}{\partial x} d\tau' \quad \text{for } t < t' \quad (29)$$

and

$$Y(\pm l, t) = Y(\pm l, t') + \frac{1}{\pi \rho l} \int_{t'}^t d\tau \int_{t'}^\tau d\tau' \frac{p(l, 0, \tau')}{\alpha h(l, 0, \tau')} \quad \text{for } t > t'$$

With increasing time, the elevation $Y(l, t)$ increases as well. A moment will be reached at which $Y(l, t)$ will be

equal to $h_0 - \int_0^t V dt$. At that moment the air layer

under the plate is cut off from the surrounding medium; and the air flow issuing from the slit will be reduced suddenly to zero.

It is highly probable that a shock wave is formed in the air layer as soon as the free escape of the air is cut off by the disturbance of the water surface at the plate edges.

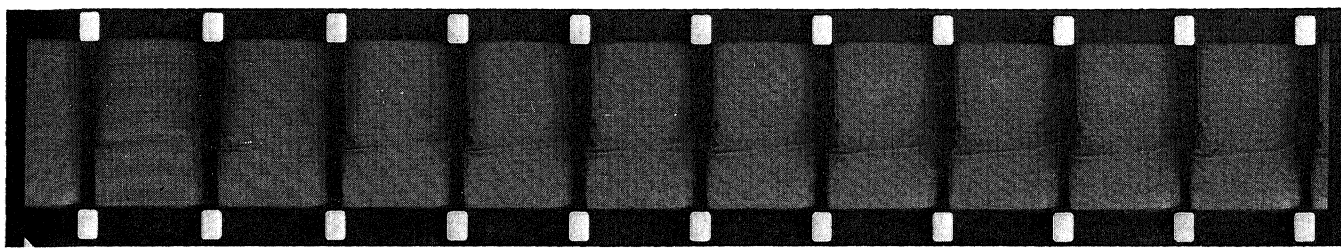


Fig. 5 Formation of bubble layer at moment of impact. The views are taken at 8000 frames/sec from beneath water surface

These shock waves travel from the edges to the center of the plate with the local sound velocity. The pressure over the shock wave is discontinuous, resulting in such a large water surface disturbance (equation 16) that the air layer is broken up into bubbles after the passage of the shock front. This is supported by experimental evidence.

Fig. 5 shows a series of pictures taken with a high-speed camera at 8000 frames/sec. The lower side of the plate is filmed from beneath the water surface. The velocity at which the bubble formation is expanding as taken from these pictures is about 350 m/sec. The calculated value of the mean sound velocity for this case is $c_{\text{mean}} = 1.08 c_0 \approx 360$ m/sec. The plate touches the water surface as soon as

$$Y(l,t) = h_0 - \int_0^t V dt.$$

The description of the slamming problem after that time needs another mathematical idealization, which will be described later in the paper.

Numerical Solution of Hyperbolic Partial Differential Equations

The quasi-linear system of equations for the dependent variables u and c_a as functions of the independent variables x and t as stated in (17) and (18) has to be solved.

With the use of equation (19) the right-hand sides of equations (17) and (18) can be expressed in terms of u , c_a , x , and t . This expression appears independent of the derivatives of u and c_a . So there are two characteristic directions

$$\begin{aligned} \frac{dx}{dt} &= u + c_a \\ \frac{dx}{dt} &= u - c_a \end{aligned} \quad (30)$$

The characteristic equations are

$$\begin{aligned} du + \frac{2}{\gamma - 1} dc_a + \frac{c_a}{h} \left(\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} \right) dt &= 0 \\ du - \frac{2}{\gamma - 1} dc_a - \frac{c_a}{h} \left(\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} \right) dt &= 0 \end{aligned} \quad (31)$$

where h is given by equation (19).

Because the boundary and initial conditions of the

problem are given on lines parallel to the axes of the x - l plane, the numerical method of specified time intervals is preferred instead of using a grid of characteristics.

The method of specified time interval starts with the assumption that u and c_a are known functions of x at a specified time $t = \bar{t}$, either as given initial conditions or as the result of a previous stage of the calculation. Using finite-difference approximations to the two partial differential equations along the characteristic directions, the values of u and c_a can be obtained at the time $t = \bar{t} + \Delta t$. The computational procedure is described by Mary Lister [11] and will not be repeated here. The boundary conditions on the lines $x = \pm l$ are stated in (26)–(28) and the initial conditions for $-l < x < l$ in (23)–(25). The correction in the pressure distribution at the edges of the plate is described in the previous section and is applied only to the calculation of the surface elevation and hence to the local distance $h(x,t)$ between plate and water.

The factor α , which is a measure for the region over which the pressure curve is smoothed at the end points, appears in these expressions as a quantity of the order unity whose exact numerical value is unknown. In the numerical computation the value of the factor α was varied. Its influence on the final result turned out to be rather small. The reason for this is connected with the following important conclusion which could be drawn from the numerical computation. The moment at which the air layer under the plate is locked appears to coincide almost with the moment that the escape velocity of the air attains the acoustic sound velocity. The influence of the numerical value of α on this phenomenon is rather small, which can be explained as follows. When the escape velocity of the air is subsonic the acceleration of the water surface under the plate edges is proportional to $\ln(\alpha h/l)$. After the acoustic sound velocity is attained the proportionality changes into $(\alpha h/l)^{-1}$ [see equation (29)]. Because $\alpha h/l \ll 1$ at the moment that $u(l,t) = c_0$, the elevation of the water surface is much more important in the latter case.

The elevation $Y(l,t)$ attains the value $h_0 - \int_0^t V dt$ just after the critical escape velocity is attained. A variation in the choice of α causes only a slight shift in this result. So the fact that the exact value of α is unknown is not so important for the practical solution of the problem.

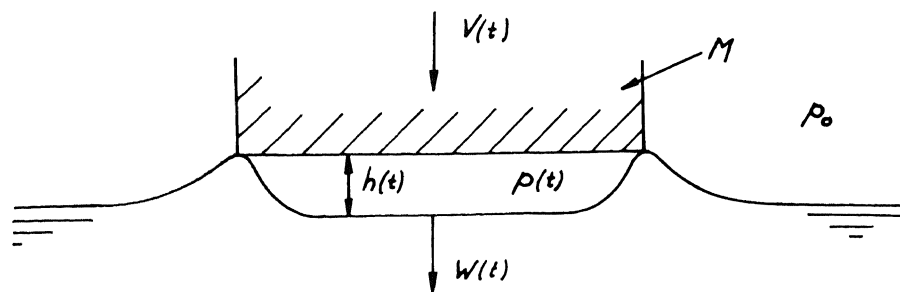


Fig. 6

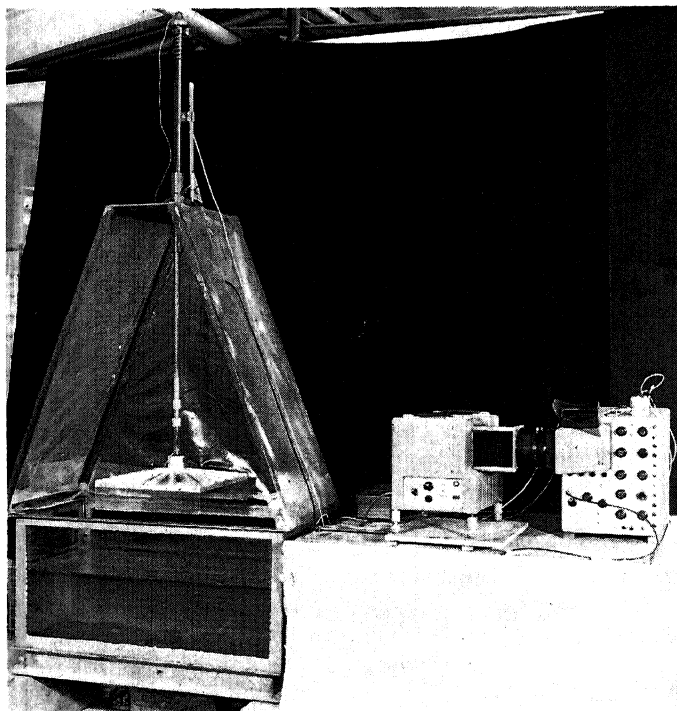


Fig. 7 Installation of test model

Compression of the Trapped Air Cushion

At last the description is given of a mathematical model for the air cushion, which elucidates its action as a buffer device. When the two-dimensional plate catches the water surface at its edges, the mean thickness of the trapped air layer is h_1 , and the mean pressure is equal to p_1 . It is assumed that from this moment the downward velocity of the water surface under the plate is only a function of time and independent of the x -coordinate. Further, the flow of air in the x -direction is neglected. The pressure in the air or bubble layer is thus place-independent. Due to the assumption that the velocity of the water surface $w(t)$ between $-1 < x < l$ is only a function of time, the virtual mass (m) of the plate will be constant and equal to the mass of a half cylinder with diameter $2l$, filled with water. The virtual mass per unit length is

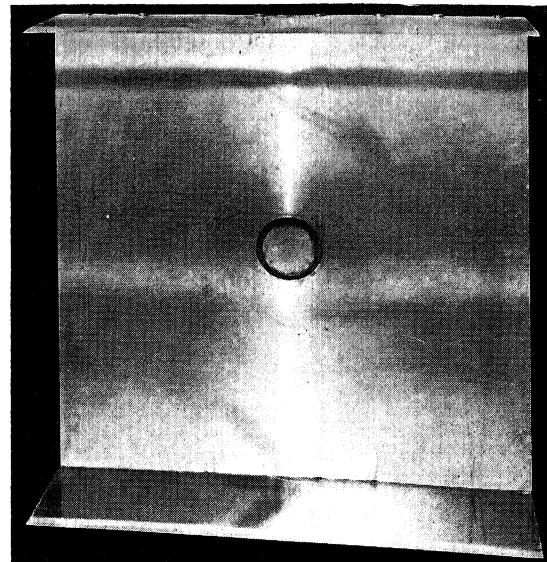


Fig. 8 Flat-plate model

$$m = \frac{\pi}{2} l^2 \rho_l \quad (32)$$

The mass of the plate (or body with flat bottom) per unit length is M ; the velocity at the moment of contact is V_0 .

In writing the equation for the plate motion it is reasonable to neglect the acceleration of gravity, because in the time intervals under consideration the weight of the plate will be small in general compared to the impact load.

$$M \frac{dV}{dt} = -2l(p - p_0) \quad (33)$$

The same equation for the virtual mass yields

$$m \frac{dw}{dt} = 2l(p - p_0) \quad (34)$$

The air compression will be isentropic, so

$$\frac{p}{p_1} = \left(\frac{\rho}{\rho_1} \right)^\gamma = \left(\frac{h_1}{h} \right)^\gamma \quad (35)$$

where γ is the ratio between the specific heat at constant pressure and that at constant volume.

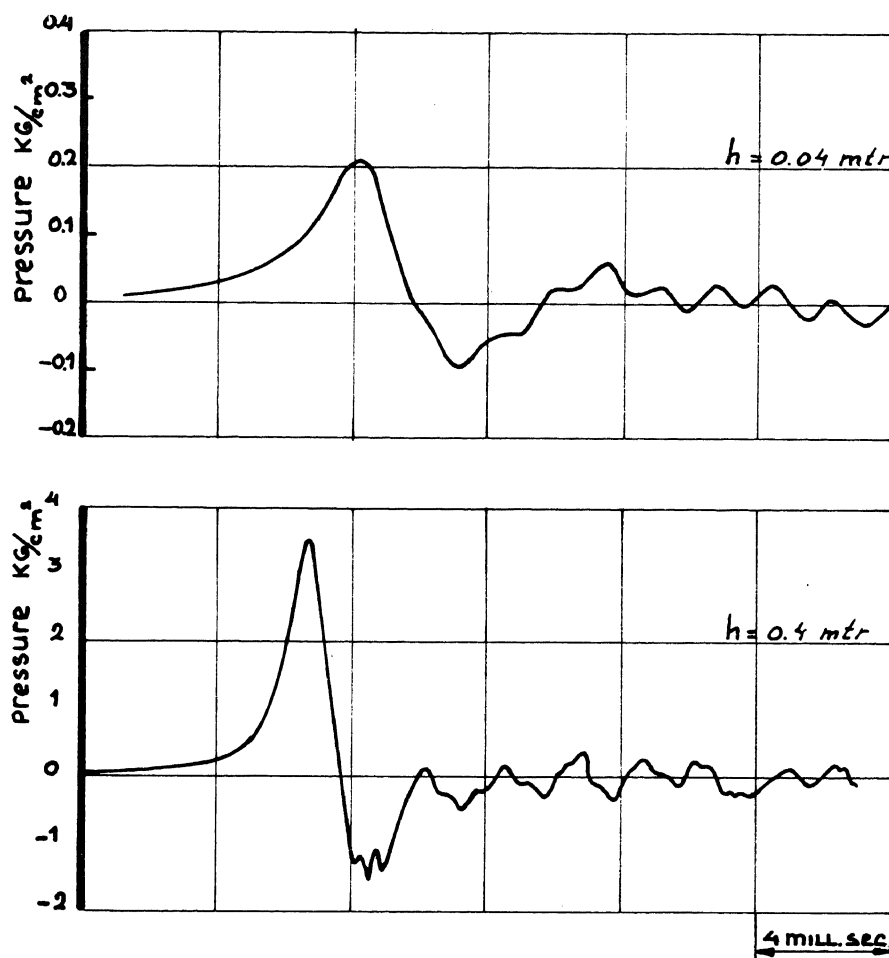


Fig. 9 Impact pressure record for test with drop height of 4 and 40 cm respectively

Using these equations, the following relation for the air layer thickness is obtained:

$$\frac{d^2h}{dt^2} = \frac{dw}{dt} - \frac{dV}{dt} = \frac{m+M}{mM} 2lp_1 \left[\left(\frac{h_1}{h} \right)^\gamma - \frac{p_0}{p_1} \right] \quad (36)$$

The initial conditions are

$$t = t_1; \quad V = V_1; \quad w = w_1 \quad (37)$$

The differential equation (36) is nonlinear.

By multiplying both sides of the equation by $2dh/dt$ and carrying out an integration over t , a linear differential equation is obtained.

$$\left(\frac{dh}{dt} \right)^2 = \frac{m+M}{mM} 4lp_1 \int_{t_1}^t \left[\left(\frac{h_1}{h} \right)^\gamma - \frac{p_0}{p_1} \right] \frac{dh}{dt} dt + (w_1 - V_1)^2$$

Evaluating the right-hand side gives

$$\frac{1}{c_0^2} \left(\frac{dh}{dt} \right)^2 = \frac{m+M}{M} \frac{4lh_1p_0}{mc_0^2} \left[-\frac{p_1}{p_0(\gamma-1)} \times \left\{ \left(\frac{h}{h_1} \right)^{1-\gamma} - 1 \right\} + 1 - \frac{h}{h_1} \right] + \left(\frac{w_1 - V_1}{c_0} \right)^2 \quad (38)$$

Using equations (32) and (35) and after rearranging, this transforms into

$$= \pm \left(\frac{p}{p_1} \right)^{\frac{1+\gamma}{\gamma}} \left[\frac{\frac{\pi}{2} l^2 \rho_l + M}{M} \cdot \frac{8\gamma l \rho_a}{\pi h_1 \rho_l} \left[\frac{p_1}{p_0(\gamma-1)} \times \left\{ 1 - \left(\frac{p}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \right\} + 1 - \left(\frac{p}{p_1} \right)^{-\frac{1}{\gamma}} \right] + \frac{\gamma^2 l^2}{h_1^2} \left(\frac{w_1 - V_1}{c_0} \right)^2 \right]^{1/2} \quad (39)$$

With the initial condition $p = p_1$ for $t = t_1$ the relation between pressure and time can be solved numerically. The impulsive pressure attains a maximum when $dp/dt = 0$. The magnitude of $p = p_{\max}$ follows from:

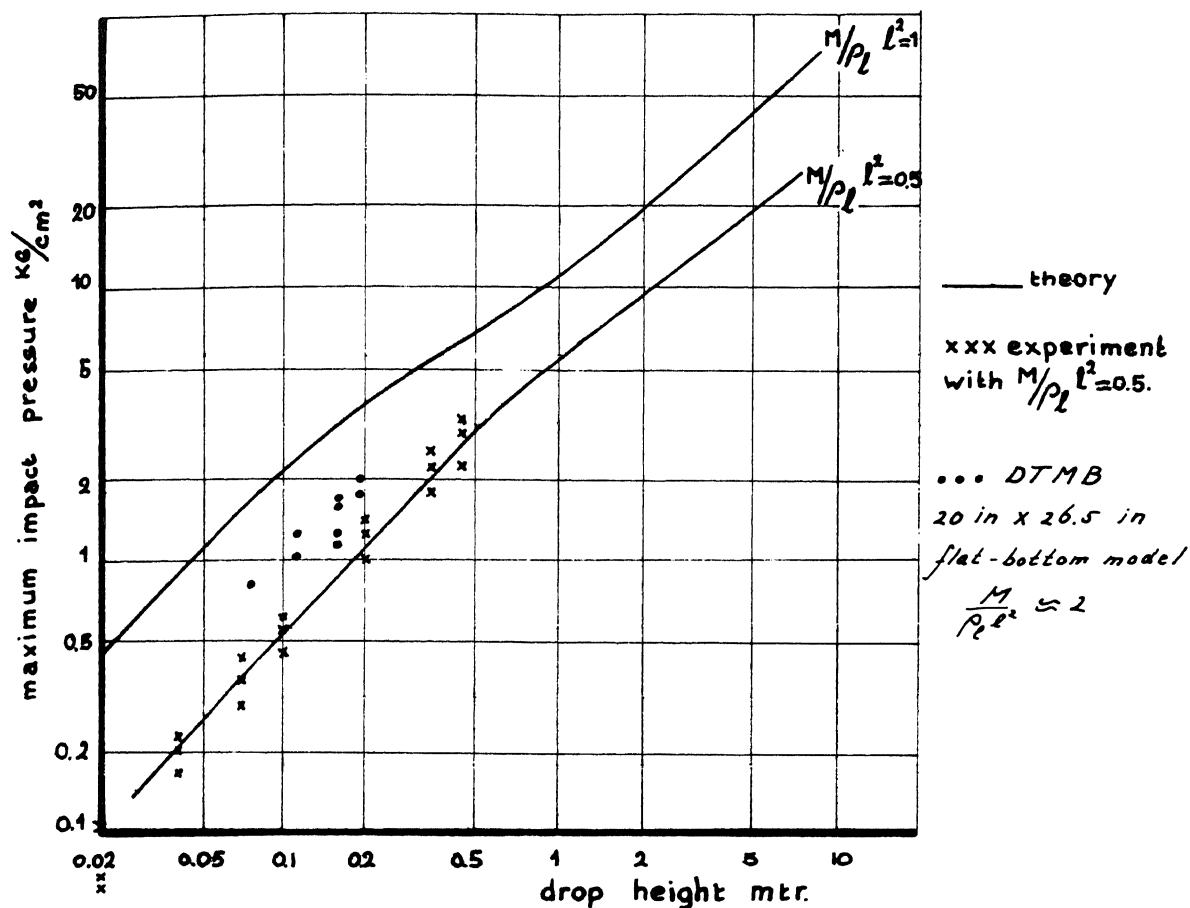


Fig. 10 Maximum impact pressure as a function of drop height

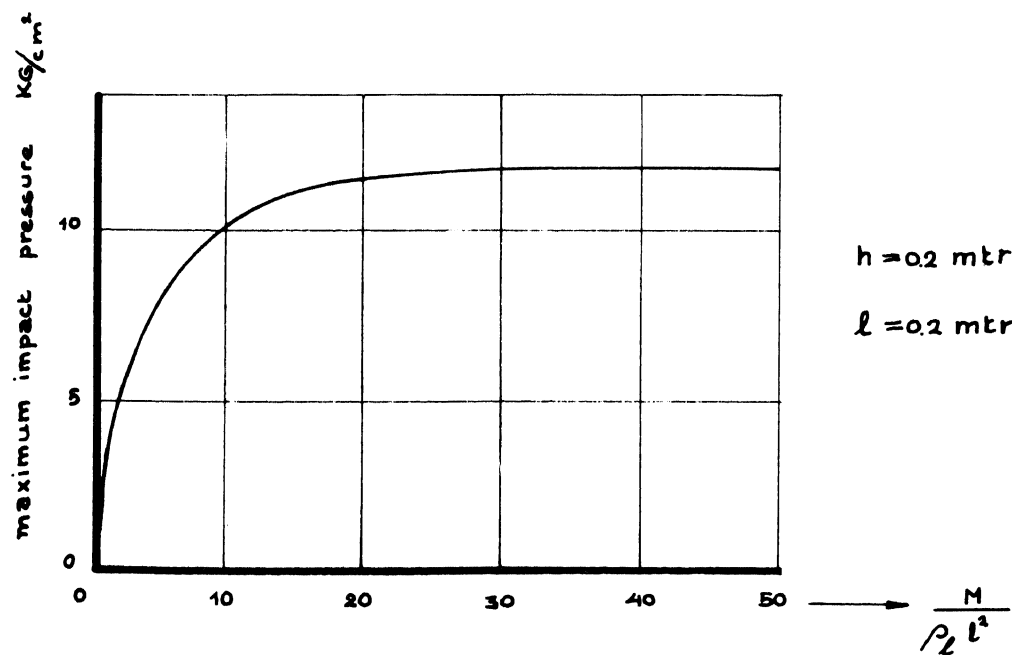


Fig. 11 Effect of falling mass on maximum impact pressure

$$\frac{1}{\gamma - 1} \frac{p_1}{p_0} \left\{ 1 - \left(\frac{p_{\max}}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \right\} + 1 - \left(\frac{p_{\max}}{p_1} \right)^{-\frac{1}{\gamma}} = - \frac{\pi}{8} \frac{\rho_l}{\rho_a} \frac{\gamma l}{h_1} \frac{M}{\frac{\pi}{2} l^2 \rho_l + M} \left(\frac{w_1 - V_1}{c_0} \right)^2 \quad (40)$$

The initial conditions of the trapped air layer h_1/l , p_1/p_0 , w_1/c_0 , and V_1/c_0 can be obtained from the calculation described in the previous sections.

The positive sign in equation (39) corresponds to a pressure increase and has to be taken before the maximum pressure is reached. Afterwards the negative sign must be used. Moreover, the right-hand side of (39) is only a function of p , for given initial conditions. Therefore, the pressure-time curve is symmetric around the time at which the maximum pressure is attained. This conclusion is in agreement with the measured pressure-time history curve.

Description of Experimental Setup

A cast aluminium plate 40 by 40 by 2 cm with bar stiffeners as shown in Fig. 7 is dropped from various heights up to 50 cm on a smooth water surface. The total weight for the test was 8 kg. The tests were carried out in an aquarium 100 by 40.5 by 40 cm with glass walls and bottom.

The support consisted of a steel frame which guided a vertical shaft, at the end of which the flat plate was attached. To simulate two-dimensional flow conditions the plate was provided with perspex vertical strips at two parallel edges; see Fig. 8. A pressure transducer of barium-titanaat crystal was fitted at the center of the plate.

By means of a photoelectric cell the point at which the plate had fallen to a few mm above the water could be marked. At that moment the time base of the oscilloscope was started and the pressure-time history photographed. Fig. 9 shows typical pressure-time history curves during a 4-cm and a 40-cm free fall. The maximum values of the impact pressure are plotted as a function of the drop height in Fig. 10.

Numerical Results and Experimental Data

The effect of the following variables on the impact pressure is numerically evaluated, where:

- h = drop height
- l = half width of the flat bottom
- M = mass of the free-falling body

Normal atmospheric conditions are chosen, namely

$$\left. \begin{aligned} g &= 9.81 \text{ m/sec}^2 \\ \rho_l &= 10^3 \text{ kg/m}^3 \\ \rho_a &= 1.29 \text{ kg/m}^3 \\ p_0 &= 10^5 \text{ N/m}^2 \\ \gamma &= 1.40 \end{aligned} \right\} \rightarrow c_a = 330 \text{ m/sec}$$

Fig. 10 presents the results for two bodies of different masses $M/\rho_l l^2 = 0.5$ and 1, respectively. The maximum

impact pressure is plotted as a function of the drop height. The half width of the flat bottom is $l = 0.2$ m. It is seen that $p_{\max}$ is approximately directly proportional to the drop height or to the square of the downward body speed. This proportionality appears to be valid for small values of $M/\rho_l l^2$. With increasing mass, however, there is a more or less linear relationship between $p_{\max}$ and V , which can be concluded from the following considerations.

The most important pressure rise under the body occurs after the moment the escape velocity of the air reaches the velocity of sound. As follows from mass continuity, the thickness of the air layer at that moment is of the order $h/l = V/c_a$. The body travels down over a distance h during a time of the order $\Delta t = h/V = l/c_a$. If $M/\rho_l l^2 \gg 1$, the body velocity is hardly decelerated by the influence of the air cushion, so the added mass of the body has to accelerate to the velocity V during the same time interval Δt .

The total impulse produced in the system will be approximately equal to the change in momentum of the added mass m alone

$$I = 2l \int_{t_0}^{t_0 + \Delta t} p dt \therefore mV$$

Hence the maximum impact pressure $p_{\max}$ is proportional to

$$p_{\max} \therefore \frac{mV}{l\Delta t} \therefore \rho_l V c_a$$

which is linear with the downward body speed.

Fig. 11 indicates that the maximum pressure is independent of the mass of the falling body when $M/\rho_l l^2 \gg 1$. The diagram is calculated for a constant drop height $h = 0.2$ m ($l = 0.2$ m). This result is qualitatively in agreement with experimental data on the impact of circular plates of varying mass on the water surface, obtained from Y. Fujita [6].

Fig. 12 shows the pressure during impact as a function of time for two different drop heights. The numerical results are compared with experimental data in Figs. 10 and 12 and the agreement is excellent

Finally, Fig. 13 shows the influence of plate dimension on the maximum impact pressure. The value of $M/\rho_l l^2 = 1$ and the drop height $h = 0.2$ m. It appears that the maximum impact pressure is nearly independent of the plate width for equal values of the ratio $M/\rho_l l^2$.

Hence, to obtain the same impact pressure of two bodies falling from an equal distance above the water surface, the mass of the body per unit bottom surface must be directly proportional to the two-dimensional body width. The larger body must be relatively heavier than the smaller one.

Discussion of Validity

The most important assumption lies in neglecting the compressibility of the water and the elasticity of the body. Compressibility effects are neglected because the events we are interested in are expected to happen in

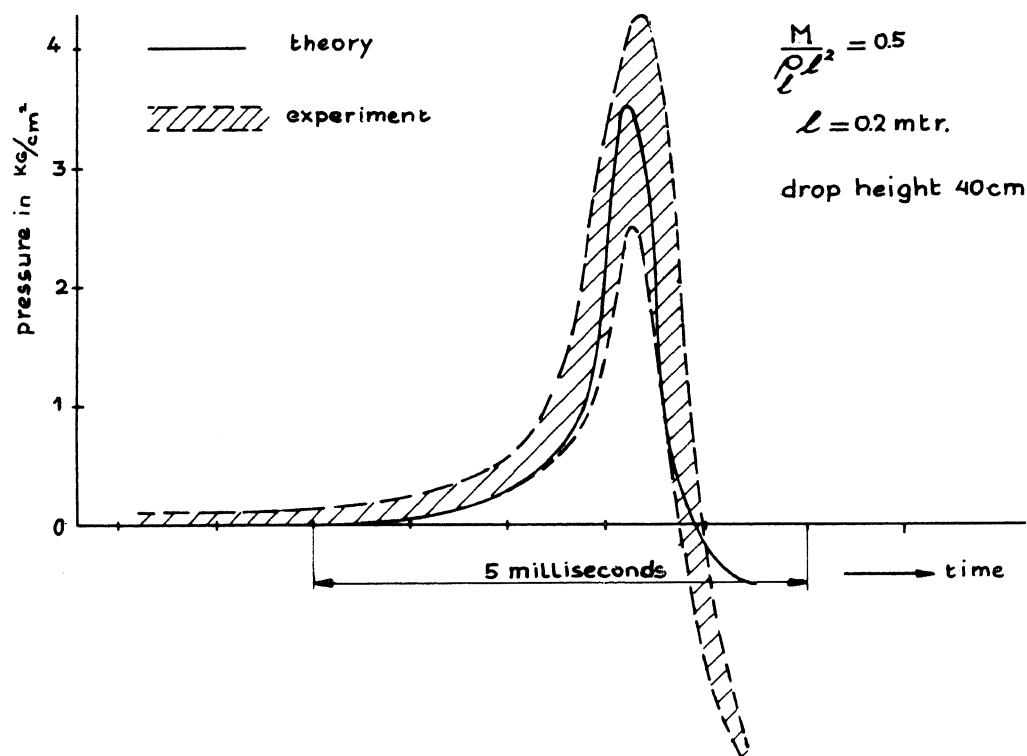
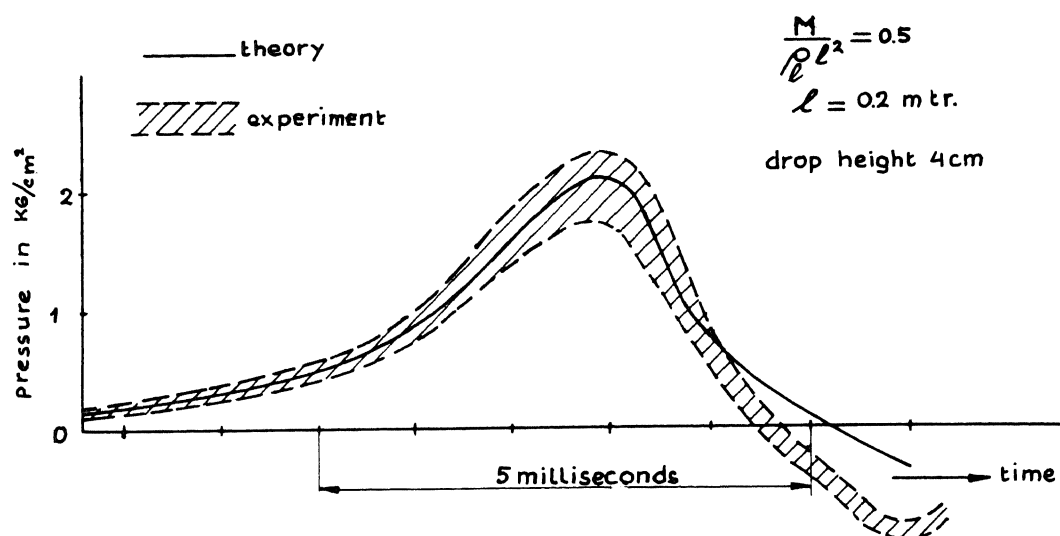


Fig. 12 Pressure-time history during impact

times of the order required by an acoustic wave in the air to travel over a distance l , i.e., $\Delta t = l/c_a$, which is large compared with l/c_l . The experiments carried out here indicate that this assumption is fully justified.

The excellent agreement between theory and experi-

ment shown in this paper may suggest that the subject of flat-bottomed-body slamming has now been exhaustively described. The experiments, however, are limited to small values of the mass of the body compared with the added mass. The author feels that the theory enunciated

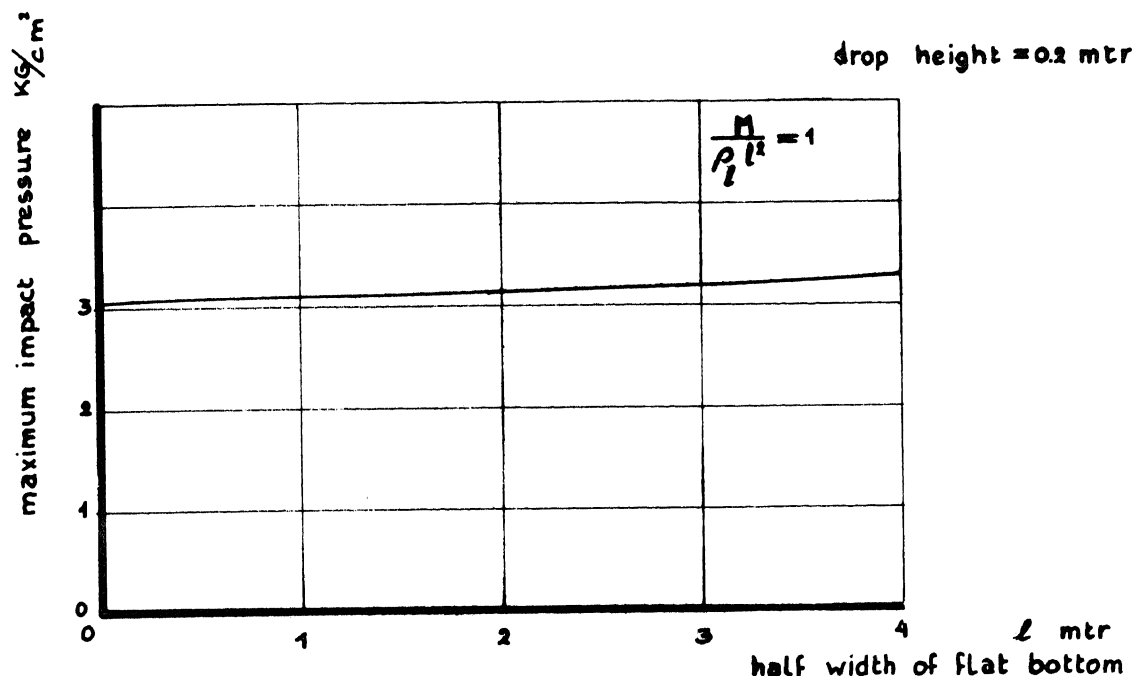


Fig. 13 Effect of plate width on maximum pressure

here will be valid only for not too high values of $M/\rho l^2$.

With increasing mass of the body the maximum impact pressure increases. When $M/\rho l^2 \gg 1$ the pressure-time curve starts approximately like $(c_a/l)(t - \tau)^{-1.4}$, where τ is the time at which the plate should reach the water surface in the absence of the air layer. With increasing value of the maximum pressure the gradient of the pressure-time curve becomes steeper. An important pressure variation can occur in a time interval small compared with c_a/l , such that the time-dependent term in the wave equation can no longer be neglected. That is, the compressibility effects of the water cannot be neglected when the maximum pressure increases. Also, the elasticity of the construction in that case must be taken into account.

A comparison of experimental data on a 20 in. by 26.5 in. flat-bottom model $M/\rho l^2 \approx 2$, obtained from Sheng-Lun Chuang [8], shows a considerably lower maximum impact pressure than predicted by theory; see Fig. 10. The discrepancy may be due to the neglected effect of water compressibility mentioned before, as well as to the possibility that the experimental model (a stiffened steel box) may not be sufficiently rigid to withstand the impulsive pressure without deflection. An indication of this possibility could be that his measured maximum impact pressure appears somewhat higher at the edges of the model than at the center, which is not to be expected [6]. Further, the impact acceleration history curve of the model shows a discrepancy in shape with the impact pressure history, which is in contradiction to Newton's second law for a rigid body. The time duration of the impact pressure is in agreement with the author's ex-

periments, which should justify also in this case our neglect of water-compressibility effects.

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